

## BOUNDARY-LAYER FLOW IN FORCED-CONVECTION FILM-BOILING ON A WEDGE

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**Abstract**—Forced convection film boiling on a wedge has been investigated by means of two-phase boundary-layer theory. It has been shown that the effect of the pressure gradient in the liquid dominates the dynamics of the flow in the vapour layer. This is in contrast to the flow past a flat plate where no appreciable pressure gradient exists. It has also been shown that the physical structure of the flow changes according as the liquid subcooling is “small” or “large”. By carefully choosing different dimensionless variables for each of these cases the physical structure of the flow was reflected in the structure of the equations. Simple analytical solutions have thus been obtained which are accurate to within a few per cent over the greater portion of the parameter range of interest. These results have been checked numerically using typical values of the parameters for a water–steam system.

### NOMENCLATURE

$f$ , dimensionless stream function in vapour;  
 $h$ , characteristic thickness of vapour film;  
 $h_0$ , actual thickness of vapour film;  
 $h_{fg}$ , enthalpy of evaporation;  
 $k$ , thermal conductivity;  
 $n$ ,  $= \beta/\pi - \beta$ , similarity number;  
 $p$ , pressure;  
 $q$ , heat transfer at the wall;  
 $u, v$ , velocity components;  
 $x, y$ , coordinates;  
 $A, B, C, D, E$ , dimensionless parameters;  
 $C_f$ ,  $= \frac{\tau_w}{\rho_l U^2} \left( \frac{2}{n+1} Re \right)^{1/2}$ ,  
 skin friction coefficient;  
 $F$ , dimensionless stream function in liquid;  
 $Q$ ,  $= \frac{Nu}{(Re)^{1/2}} \left( \frac{2}{n+1} \right)^{1/2}$ ,  
 heat-transfer coefficient;  
 $Nu$ ,  $= \frac{qx}{k_v \Delta T_v}$ , Nusselt number;  
 $Pr$ , Prandtl number;  
 $R$ ,  $= V/U$ , velocity ratio;  
 $S$ ,  $= h/\delta$ , thickness ratio;  
 $T$ , temperature;  
 $U$ ,  $\sim x^n$ , external velocity;  
 $V$ , characteristic vapour velocity;  
 $\Delta T_v$ , temperature difference in vapour;  
 $\Delta T_l$ , temperature difference in liquid.

$\nu$ , kinematic viscosity;  
 $\rho$ , density;  
 $\varphi$ , dimensionless temperature in liquid;  
 $\xi$ , similarity variable in liquid;  
 $\tau_w$ , skin friction at wall;  
 $\psi$ , stream function in vapour;  
 $\Psi$ , stream function in liquid.

### Subscripts

$l$ , liquid property;  
 $v$ , vapour properties;  
 $1$ , small subcooling parameter;  
 $2$ , large subcooling parameter.

### INTRODUCTION

INVESTIGATIONS of film boiling are usually concerned with one of the two cases of forced or free convection. In forced convection the motion is generated by an oncoming stream of liquid and it is assumed that buoyancy forces are negligible, whereas in free convection there is no prescribed flow at large distances from the heated surfaces and the motion is generated by the buoyancy forces within the vapour and liquid layers. We consider here the former case, that of forced convection.

Forced convection over a flat plate has been analysed previously by Cess and Sparrow [1] using a combination analytical–numerical method. They showed that if the superheating is not too large the inertia terms in the vapour phase may be neglected and hence the vapour equations linearised, but they found it necessary to use numerical results for the liquid phase in order to determine a boundary condition on the vapour at the interface. Their work has been extended by Ito and Nishikawa [2] to cover a larger range of temperatures though this necessitated solving the equations in both phases numerically. For flow past a flat plate there is of course no appreciable pressure gradient in the liquid.

### Greek symbols

$\alpha$ , thermal diffusivity;  
 $\beta$ , wedge half-angle;  
 $\delta$ , liquid boundary-layer thickness;  
 $\eta$ , similarity variable in vapour;  
 $\eta_0$ , dimensionless height of interface;  
 $\theta$ , dimensionless temperature in vapour;  
 $\mu$ , absolute viscosity;

For bodies of any finite thickness, however, pressure gradients will exist in the liquid and when the flow is of boundary-layer type this pressure gradient will be transmitted to the vapour layer, where its effect will be greatly enhanced because of the large density ratio. It will be shown that this effect dominates the dynamics of the vapour layer, and for this reason the flat-plate solution should be regarded as somewhat over-idealised.

The significance of pressure gradients in the liquid has not been properly evaluated in the published literature; for example, Hsiao *et al.* [3], who considered flow past a sphere, erroneously omitted the relevant term completely. It seems desirable, therefore, to begin by considering the simple case of flow past a wedge for which similarity solutions are available. (See Fig. 1 for a sketch of the coordinate system.)

A further contribution made in the present paper is to explain the advantages of a careful choice of dimensionless variables so as to reflect the true physical structure of the flow. No attempt to do this was made in [1] or [2], and although the solutions obtained by those authors are formally correct they depend heavily on computed graphs and tables. By contrast we show that in two limiting cases, corresponding (roughly speaking) to small and large values of the liquid subcooling, simple analytical solutions can be found which are accurate to within a few per cent over the greater portion of the parameter range of interest.

Typical values of parameters have been given for a water steam system.

#### CONSERVATION EQUATIONS AND BOUNDARY CONDITIONS

It is assumed the boundary-layer equations hold in the liquid phase near the interface:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_l} \frac{\partial p}{\partial x} + \nu_l \frac{\partial^2 u}{\partial y^2}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_l \frac{\partial^2 T}{\partial y^2}. \quad (3)$$

It is assumed that the properties of the liquid and vapour are constant. The pressure gradient in (2) is determined by the flow at the outer edge of the boundary layer and is constant across it:

$$-\frac{1}{\rho_l} \frac{\partial p}{\partial x} = U U_x. \quad (4)$$

At the interface between the liquid and vapour it is required that the following quantities should be continuous: (a) mass flow crossing the interface, (b) tangential velocity, (c) tangential stress, (d) normal stress, (e) temperature; there must also be an energy balance. Within the boundary-layer approximation

these are equivalent to

$$\rho_l \left( u \frac{dh_0}{dx} - v \right)_{\text{liq}} = \rho_v \left( u \frac{dh_0}{dx} - v \right)_{\text{vap}} \quad (5)$$

$$u_{\text{liq}} = u_{\text{vap}} \quad (6)$$

$$\mu_l \left( \frac{\partial u}{\partial y} \right)_{\text{liq}} = \mu_v \left( \frac{\partial u}{\partial y} \right)_{\text{vap}} \quad (7)$$

$$p_{\text{liq}} = p_{\text{vap}} \quad (8)$$

$$T_{\text{liq}} = T_{\text{vap}} = T_s \quad (9)$$

$$\begin{aligned} \rho_v \left( u \frac{dh_0}{dx} - v \right)_{\text{vap}} \\ = \frac{1}{h_{fg}} \left[ k_l \left( \frac{\partial T}{\partial y} \right)_{\text{liq}} - k_v \left( \frac{\partial T}{\partial y} \right)_{\text{vap}} \right]. \end{aligned} \quad (10)$$

In the vapour it is also assumed that the boundary-layer equations hold, hence we have (1) and (3) with vapour properties replacing liquid properties. However equation (8) requires that pressure is continuous at the interface, so that equation (2) becomes

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\rho_l}{\rho_v} U U_x + \nu_v \frac{\partial^2 u}{\partial y^2}. \quad (11)$$

The boundary conditions at the wedge surface and in the undisturbed flow are

$$u = v = 0, \quad T = T_0 \quad \text{at } y = 0 \quad (12)$$

$$u = U, \quad T = T_\infty \quad \text{at } y = \infty. \quad (13)$$

#### SIMILARITY TRANSFORMATIONS

The problem of selecting suitable dimensionless similarity variables now arises; this is done so that the velocity and temperature fields are described by order 1 dimensionless functions.

It is expected that the liquid velocity will be of the same order of magnitude as the free stream velocity, hence the following variables are defined for the liquid phase:

$$\xi = y/\delta, \quad \Psi = U\delta F(\xi), \quad \varphi(\xi) = \frac{T - T_s}{\Delta T_l}. \quad (14)$$

Here  $\Psi$  is the stream function so that  $u = UF'$  and  $F'$  will be an order 1 quantity as required,  $\delta$  is the liquid boundary-layer thickness

$$\delta = \left( \frac{2}{n+1} \frac{\nu_l x}{U} \right)^{1/2}. \quad (15)$$

Substituting into (2) and (3) it follows that

$$F''' + FF'' + \frac{2n}{n+1} (1 - F'^2) = 0 \quad (16)$$

and

$$\varphi'' + Pr_1 F \varphi' = 0. \quad (17)$$

However in the vapour phase the term due to the pressure gradient is very large because of the low density of the vapour compared with that of the

liquid; there is therefore no reason to expect the vapour velocity to be of order  $U$ . Since the viscous term in the vapour momentum equation (13) must be important for the no slip condition to be satisfied at the wedge surface a balance is set up between the pressure gradient and viscous terms. If the vapour velocity is of order  $V$  and the film thickness is of order  $h$ , this balance gives

$$O\left(\frac{\rho_l U^2}{\rho_v x}\right) = O\left(\frac{\rho_v V}{h^2}\right). \quad (18)$$

An examination of equation (10) now shows that the heat transferred from the wedge surface to the interface may be accounted for in one of two ways (a) vaporisation of the liquid or (b) conduction through the liquid.

The cases when either (a) or (b) is dominant are now considered.

(i) *Small subcooling*

Suppose first that (a) is dominant, then the subcooling is small. In order for the relevant terms in (10) to balance it is necessary that

$$O\left(\frac{k_v \Delta T_v}{h_{fg} h}\right) = O\left(\frac{\rho_v V h}{x}\right), \quad (19)$$

where use has been made of the continuity equation (1) to determine the order of  $v$ .

Eliminating  $h^2/x$  between (18) and (19) yields

$$\frac{V^2}{U^2} = \frac{\rho_l k_v \Delta T_v}{\rho_v \mu_v h_{fg}} = R_1^2 \approx 2\Delta T_v, \quad (20)$$

and (15), (18) and (19) give

$$\frac{h}{\delta} = \left(\frac{\mu_v}{\mu_l} R_1\right)^{1/2} = S_1 \approx 0.3\Delta T_v^{1/4} \quad (21)$$

where values of the coefficients have been determined for a water–steam system. Note that for typical values of  $\Delta T_v$ , the order of  $V$  will be much larger than  $U$  as was anticipated.

The following variables are therefore defined for the vapour phase:

$$\eta = y/h, \quad \psi = Vhf(\eta), \quad \theta(\eta) = \frac{T - T_s}{\Delta T_v}. \quad (22)$$

Then  $u = \partial\psi/\partial y = Vf'$  and hence  $f'$  will be order 1 as required. Substituting into (13) and the vapour phase equivalent of (3) it follows that

$$f''' + \frac{2n}{n+1} = A_1 \left( \frac{2n}{n+1} f'^2 - ff'' \right) \quad (23)$$

$$\theta'' + A_1 Pr_v f\theta' = 0. \quad (24)$$

It remains to determine the boundary conditions (5)–(12) in terms of the new variables. (Note that since  $\xi$  does not appear explicitly in the equations or boundary conditions the interface can be taken at  $\xi = 0$  without any loss of generality.) These become

$$F(0) = B_1 f(\eta_0), \quad f'(\eta_0) = C_1 F'(0) \quad (25), (26)$$

$$F''(0) = D_1 f''(\eta_0),$$

$$f(\eta_0) + \theta'(\eta_0) = E_1 \varphi'(0) \quad (27), (28)$$

$$\theta(0) = \varphi(0) = 1, \quad \theta(\eta_0) = \varphi(\infty) = 0 \quad (29)$$

$$f(0) = f'(0) = 0, \quad F'(\infty) = 1. \quad (30)$$

The boundary conditions have been presented in the form in which they will be applied; thus if the quantities on the RHS of the equations are known then the quantities on the LHS are the given boundary conditions.

*Simplification.* The dimensionless variables have been constructed so that  $E_1$  is small when the subcooling is small:

$$E_1 = R_1^{1/2} \left[ \frac{\mu_v}{\mu_l} \left( \frac{k_l \Delta T_l}{k_v \Delta T_v} \right)^2 \right]^{1/2} \approx 5.8 \frac{\Delta T_l}{\Delta T_v^{3/4}} \quad (31)$$

Hence if  $\Delta T_l$  is sufficiently small we may neglect the RHS of equation (28). The other parameters are

$$A_1 = \frac{\rho_v}{\rho_l} R_1^2 \approx 8.8 \times 10^{-4} \Delta T_v \quad (32)$$

$$B_1 = \frac{\rho_v}{\rho_l} R_1 S_1 \approx 2 \times 10^{-4} \Delta T_v^{3/4} \quad (33)$$

$$C_1 = R_1^{-1} \approx 0.7 \Delta T_v^{-1/2} \quad (34)$$

$$D_1 = S_1 \approx 0.3 \Delta T_v^{1/4}, \quad (35)$$

where again the values of coefficients have been given for a water–steam system. Note that  $\Delta T_l$  does not appear in any of the parameters  $A_1, B_1, C_1, D_1$  since there are no significant temperature variations in the liquid.

The vapour phase is coupled to the liquid boundary layer only through the boundary condition applied at the interface. However  $C_1$  is small and for  $\Delta T_v$  sufficiently large the RHS of (28) is negligible. It is then not necessary to determine the liquid flow in order to solve the vapour equations, although having solved the vapour equations it is then possible to substitute back into the boundary conditions (25) and (27) and hence solve the liquid equations.

The physical situation is that the main body of the vapour is travelling at such a speed relative to the liquid that its velocity at the interface may be neglected to a first approximation; this is due to the effect of the large pressure gradient in the vapour.

Further, if  $A_1$  is small the non-linear terms in (23) and (24) will be negligible and the solution of the vapour equations may be obtained in a closed form. Hence

$$\eta_0 = \left( \frac{6\pi}{\beta} \right)^{1/4}. \quad (36)$$

This gives us the skin friction and heat-transfer coefficients

$$C_f = D_1 \frac{\beta}{\pi} \eta_0, \quad Q = \frac{1}{D_1 \eta_0}. \quad (37)$$

In order to check these results (16), (17), (23) and (24) with boundary conditions (25)–(30) have been solved numerically at a wedge half angle of  $\pi/4$  for  $\Delta T_v = 200$  and  $\Delta T_v = 500$ . (For numerical work the values  $Pr_v = 0.996$  and  $Pr_l = 1.74$  were used.)

The analytic solution (36) is  $\eta_0 = 2.21$  whereas the numerical solutions give  $\eta_0 = 2.098$  for  $\Delta T_v = 200$  and  $\eta_0 = 2.099$  for  $\Delta T_v = 500$  (in both cases  $\Delta T_l = 0$ ). There may be two reasons for the discrepancy, (i) the neglect of the inertia terms, or (ii) the neglect of the velocity at the interface. In order to find the next order terms a perturbation analysis may be carried out in each case, in (i) by taking  $A_1$  as a small parameter, and in (ii) by perturbing about the potential flow solution  $F = \xi$ . Boundary conditions on  $F$  may be obtained using the analytic solution for  $f(\eta_0)$  and  $f''(\eta_0)$  in (25) and (27); however  $B_1$  is very small so that  $F(0) = 0$  may be taken. It is found that the effects of (ii) are much greater than the effects of (i) and incorporating this correction then to leading order in the small quantity  $C_1$

$$\eta_0 = \left(\frac{6\pi}{\beta}\right)^{1/4} \left[1 - \frac{6^{1/2}}{4} F'(0)C_1\right] \quad (38)$$

where

$$F'(0) = 1 + \frac{D_1}{(8\pi)^{1/2}} \left(\frac{6\pi}{\beta}\right)^{1/4}.$$

This yields the estimate  $\eta_0 = 2.11$  in satisfactory agreement with the numerical solution.

The numerical solution for  $\eta_0$  decreases rapidly when subcooling is introduced and examination of equation (28) shows that this is to be expected. For example, at  $\Delta T_v = 200$ ,  $\Delta T_l = 3.5$  the numerical solution gives  $\eta_0 = 1.81$ . Obviously it is possible to solve the equations numerically for any value of  $\Delta T_l$ . However as  $\Delta T_l$  increases it is apparent that the physical situation changes, the liquid conduction term soon dominates equation (10) and the vapour film thickness changes in order to accommodate the given subcooling in the liquid. A different choice of dimensionless variables would therefore seem suitable.

(ii) *Large subcooling*

Most of the heat arriving at the interface is now conducted away through the liquid. Hence

$$O\left(\frac{k_v \Delta T_v}{h}\right) = O\left(\frac{k_l \Delta T_l}{\delta}\right). \quad (39)$$

It is still necessary to maintain the balance given by equation (18), and  $O(\delta)$  is determined by equation (15). Equations (39), (15) and (18) imply

$$\frac{V}{U} = \frac{\mu_l}{u_v} \left(\frac{k_v \Delta T_v}{k_l \Delta T_l}\right)^2 = R_2 \approx \frac{1}{24} \left(\frac{\Delta T_v}{\Delta T_l}\right)^2 \quad (40)$$

$$\frac{h}{\delta} = \frac{k_v \Delta T_v}{k_l \Delta T_l} = S_2 \approx 0.05 \frac{\Delta T_v}{\Delta T_l}. \quad (41)$$

Note that the order of  $V$  is dependent on  $\Delta T_l$  for a given  $\Delta T_v$  and so will not in general be of order  $U$ .

Defining similarity variables in the same form given by equation (22) and substituting into the momentum, energy equations and boundary conditions, equations (23)–(30) are obtained except that equation (28) is replaced by

$$\varphi'(0) - \theta'(\eta_0) = E_2 f(\eta_0) \quad (42)$$

and  $A_1, B_1, C_1, D_1$  are replaced by  $A_2, B_2, C_2, D_2$ .

*Simplification.* The variables have been constructed so that  $E_2$  will be small if  $\Delta T_l$  is large:

$$E_2 = \frac{\rho_v}{\rho_l} \frac{\mu_v h_{fg}}{k_v \Delta T_v} R_2^2 \approx 9 \times 10^{-4} \frac{\Delta T_v^3}{\Delta T_l^4}. \quad (43)$$

Hence if  $\Delta T_l$  is sufficiently large we may neglect the RHS of equation (42). The other parameters are

$$A_2 = \frac{\rho_v}{\rho_l} R_2^2 \approx 7.8 \times 10^{-7} \left(\frac{\Delta T_v}{\Delta T_l}\right)^4 \quad (44)$$

$$B_2 = \frac{\rho_v}{\rho_l} R_2 S_2 \approx 9.4 \times 10^{-7} \left(\frac{\Delta T_v}{\Delta T_l}\right)^3 \quad (45)$$

$$C_2 = R_2^{-1} \approx 24 \left(\frac{\Delta T_l}{\Delta T_v}\right)^2 \quad (46)$$

$$D_2 = S_2 \approx 0.05 \frac{\Delta T_v}{\Delta T_l}. \quad (47)$$

The situation is not as in case (i) where it was possible to solve the vapour equations without determining the liquid boundary-layer flow, since the solution is now dependent on the temperature gradient in the liquid via (42). It is necessary to know the liquid flow in order to determine this gradient.

If potential flow is assumed in the liquid, then  $F(\xi) = \xi$  and equation (17) may be solved analytically yielding

$$\varphi'(0) = -\left(\frac{2Pr_l}{\pi}\right)^{1/2}. \quad (48)$$

For  $\Delta T_l$  sufficiently large  $A_2$  is small and the non-linear terms in equation (24) may be neglected. In this case the temperature gradient in the vapour may be found independently of the vapour velocity, and thus

$$\eta_0 = \left(\frac{\pi}{2Pr_l}\right)^{1/2}. \quad (49)$$

The skin friction and heat-transfer coefficients are given by

$$C_f = D_2 \frac{\beta}{\pi} \eta_0, \quad Q = \frac{1}{D_2 \eta_0}. \quad (50)$$

These results depend on the assumption of potential flow in the liquid; it is therefore necessary to decide when this is valid. In the range of temperatures being considered  $B_2$  is extremely small and hence the RHS of equation (25) may be neglected. The tangential stress on the liquid at the interface has also been neglected although equation (47) shows that  $D_2$  is not very small until  $\Delta T_l$  is very large. However  $f''(\eta_0)$  is numerically small in the range of interest. Assuming potential flow in the liquid and assuming  $A_2$  to be small then

$$f''(\eta_0) = -\frac{2n}{n+1} \frac{\eta_0}{2} + \frac{C_2}{\eta_0}. \quad (51)$$

Thus  $f''(\eta_0)$  is small and the assumption of potential flow valid for a range of temperatures such that  $C_2$  is close to  $\beta/2Pr_l$ . In fact for  $\beta = \pi/4$ ,  $f''(\eta_0)$  will vanish when  $\Delta T_l \approx 19$  for  $\Delta T_v = 200$  and  $\Delta T_l \approx 48$  for  $\Delta T_v = 500$ .

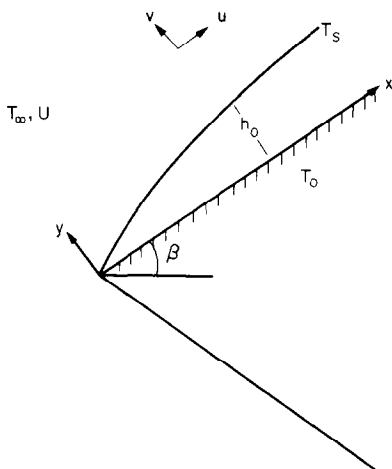


FIG. 1.

In order to check these results equations (16), (17), (23) and (24) with boundary conditions (25), (26), (27), (42), (29), (30) (replacing  $A_1$  by  $A_2$  etc.) have been solved numerically at a wedge half angle of  $\pi/4$  for  $\Delta T_v = 200$  and  $\Delta T_v = 500$  over the range  $5 \leq \Delta T_l \leq 95$  at steps of  $5^\circ\text{C}$ . The results are presented in Figs. 2 and 3. It can be seen that the analytic results agree with the numerical results to within a few percent for  $\Delta T_l > 10$  when  $\Delta T_v = 200$  and for  $\Delta T_l > 20$  when  $\Delta T_v = 500$ .

Note that it would appear that as  $\Delta T_l$  becomes very large  $R_2$  becomes small and  $C_2$  large, and therefore a further change of variables would be appropriate. This is indeed the case; however no simplifications occur which would enable a closed solution to be determined as the liquid flow is not close to potential flow and it is still necessary to find

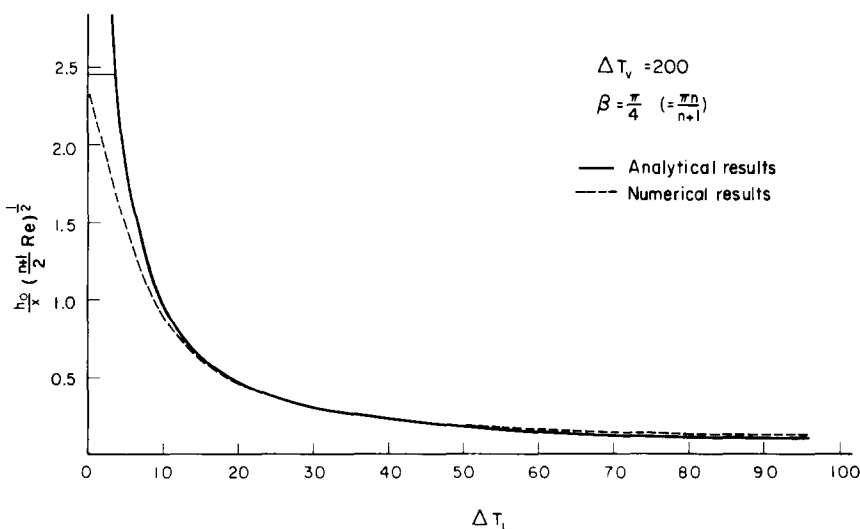


FIG. 2. Comparison of analytical and numerical results for vapour film thickness at  $\Delta T_v = 200$ ,  $\beta = \pi/4$ .

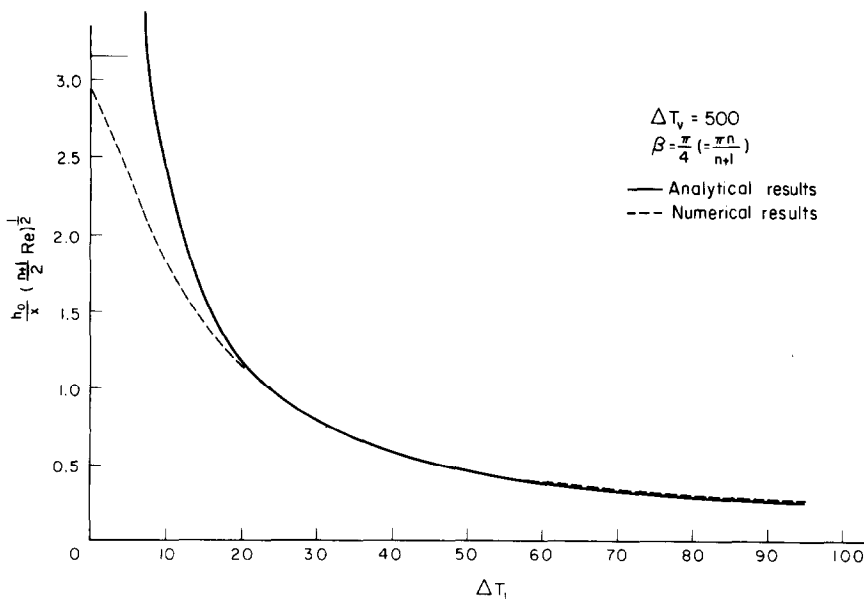


FIG. 3. Comparison of analytical and numerical results for vapour film thickness at  $\Delta T_v = 500$ ,  $\beta = \pi/4$ .

$\phi'(0)$ . The physical reason why this happens is that as  $\Delta T_i$  increases the vapour layer becomes very thin, so a large tangential stress is exerted on the liquid at the interface.

#### CONCLUSION

It has been shown that within the two regimes of small and large subcooling values for the skin friction and heat-transfer coefficients may be obtained by an analytic procedure. These coefficients have been checked numerically for two typical values of superheating at a wedge half angle of  $\pi/4$  for a water-steam system and have found to be accurate to within a few per cent except for the range of subcooling where no term is negligible in the energy balance at the interface. This range is small—of the order of 10–20°C for the values of superheating we have considered.

It has been shown that to a good approximation potential flow may be assumed in the liquid phase since for small subcooling it is not necessary to

determine the liquid boundary-layer flow and for large subcooling the tangential stress exerted on the liquid at the interface by the vapour is small.

Numerical solutions were obtained using the facilities at UMRCC by means of the NAG library routine DØ2AGA which solves a two point boundary value problem by a Runge-Kutta method and Newton iteration.

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#### ÉCOULEMENT DE COUCHE LIMITE DANS LA CONVECTION FORCÉE AVEC EBULLITION EN FILM SUR UN COIN

**Résumé**—La convection forcée avec ébullition en film sur un coin a été étudiée à partir de la théorie de couche limite diphasique. On montre que l'effet du gradient de pression dans le liquide domine la dynamique de l'écoulement sur une plaque plane où il n'existe pas de gradient de pression appréciable. On montre aussi que la structure physique de l'écoulement change selon que le sous-refroidissement du liquide est "faible" ou "fort". En choisissant convenablement différentes variables sans dimension pour chacun de ces cas, la structure de l'écoulement se réfère dans la structure des équations. On a obtenu des solutions analytiques exactes à quelques pourcents sur la majeure portion du domaine intéressant. Ces résultats ont été testés numériquement en prenant des valeurs typiques des paramètres d'un système vapeur-eau.

#### GRENZSCHICHT-STRÖMUNG BEI FILMSIEDEN MIT ERZWUNGENER KONVEKTION AN EINEM KEIL

**Zusammenfassung**—Es wurde Filmsieden an einem Keil bei erzwungener Konvektion mittels der Zwei-Phasen-Grenzschichttheorie untersucht. Es zeigte sich, daß der Einfluß des Druckgradienten in der Flüssigkeit die Strömungsdynamik in der Dampfschicht bestimmt. Hier besteht ein Unterschied zur Strömung an einer ebenen Platte, bei der kein merklicher Druckabfall auftritt. Weiter wurde gezeigt, daß die physikalische Struktur der Strömung wechselt, je nachdem die Flüssigkeitsunterkühlung "klein" oder "groß" ist. Durch sorgfältige Wahl verschiedener dimensionsloser Variablen für jeden dieser Fälle wurde die Struktur der Strömung im Aufbau der Gleichungen wiedergegeben. So erhielt man einfache analytische Lösungen, die innerhalb einer Fehlergrenze von wenigen Prozent über der größeren Teil des interessierenden Parameterbereichs genau sind. Diese Resultate wurden numerisch geprüft, wobei typische Werte der Parameter für Wasser benutzt wurden.

#### ТЕЧЕНИЕ В ПОГРАНИЧНОМ СЛОЕ НА ПОВЕРХНОСТИ КЛИНА ПРИ НАЛИЧИИ ПЛЁНОЧНОГО КИПЕНИЯ И ВЫНУЖДЕННОЙ КОНВЕКЦИИ

**Аннотация**—С помощью теории двухфазного пограничного слоя исследовано плёночное кипение на поверхности клина при наличии вынужденной конвекции. Показано, что градиент давления в жидкости оказывает большее влияние на процесс, чем динамика течения в паровом слое. Этот вывод отличается от случая обтекания плоской пластины, где градиент давления практически отсутствует. Показано также, что в зависимости от величины недогрева жидкости меняется физическая структура потока. В каждом из этих случаев безразмерные переменные выбраны таким образом, что структура уравнений отражает физическую структуру потока. Получены простые аналитические решения, дающие погрешность в несколько процентов для представляющей интерес большей части диапазона изменения параметра. Эти результаты проверены численно при использовании типичных значений параметров для паро-водяной системы.